The Variability of Ground-Motion Prediction Models and its Components

Linda Al Atik\(^1\), Norman Abrahamson\(^2\), Julian Bommer\(^3\), Frank Scherbaum\(^4\), Fabrice Cotton\(^5\), and Nicolas Kuehn\(^4\)

1. PEER Center, University of California at Berkeley, Berkeley, CA, USA
2. Geosciences Department, PG&E, 245 Market Street, San Francisco, CA, USA
3. Civil & Environmental Engineering, Imperial College London, London SW7 2AZ, UK
4. Inst. Erd- und Umweltwissenschaften Universität Potsdam, P.O. Box 601533, D-14415, Potsdam, Germany
5. LGIT, Université Joseph Fourier, CNRS, BP 53, F-38041, Grenoble, France

Introduction

Modern ground-motion prediction models use datasets of recorded ground-motion parameters at multiple stations during different earthquakes and in various source regions to generate equations that are later used to predict site-specific ground motions. These models describe the distribution of ground motion in terms of a median and a logarithmic standard deviation (e.g., Strasser et al., 2009). This standard deviation, generally referred to as sigma (\(\sigma\)), exerts a very strong influence on the results of probabilistic seismic hazard analyses (PSHA) (e.g., Bommer and Abrahamson, 2006). Although there are numerous examples of sigma being neglected in seismic hazard, it is now generally accepted that integration over the full distribution of ground motions is an indispensable element of PSHA (Bommer and Abrahamson, 2006). Attempts to justify, on a statistical basis, a truncation of the ground-motion distribution at a specified number of standard deviations above the median have proven unfeasible with current strong-motion datasets (Strasser et al., 2008). The most promising approach to reduce the overall impact of sigma on the results of PSHA is to find legitimate approaches to reduce the value of the standard deviation associated with ground-motion prediction equations (GMPEs).

The present state-of-the-practice of seismic hazard studies applies the standard deviations from ground-motion models developed using a broad range of earthquakes, sites, and regions to analyze the hazard at a single site from a single small source region. Such practice assumes that the variability in ground motion at a single site-source combination is the same as the variability in ground motion observed in a more global dataset and is referred to as the ergodic assumption (Anderson and Brune, 1999).

\(^{1}\) Corresponding author: Email: l_atik@berkeley.edu
In recent years, the availability of well recorded ground motions at single sites from multiple occurrences of earthquakes in the same regions allowed researchers to estimate the ground-motion variability without including the ergodic assumption. It has been observed in previous studies (e.g., Lin et al., 2010; Chen and Tsai, 2002; Atkinson, 2006; Morikawa et al., 2008; Anderson and Uchiyama, 2010) that removing the ergodic assumption leads to a smaller variability of the ground motion; however, removing the ergodic assumption from the variability of the ground-motion model also requires removing it from its median resulting in the need for site-specific and path-specific ground-motion models. In the absence of data to constrain such models, removing the ergodic assumption from seismic hazard results in increased epistemic uncertainty in the median ground motion for a single site-path combination. This epistemic uncertainty is manifested by additional branches in the ground-motion logic tree.

The key to reducing the aleatory sigma is identifying those components of ground-motion variability at a single site that are repeatable rather than purely random, so that these may be removed from the aleatory variability and transferred to the quantification of the epistemic uncertainty. To this end, in this paper we revisit the basics of ground-motion regression models and break down the residuals and the variability of the models into their respective components to provide a clear understanding of the uncertainty in seismic hazard studies. Breaking down the variability of ground-motion models is a fundamental step in characterizing the uncertainty and applying the non-ergodic assumption in seismic hazard studies (e.g., Walling, 2009).

To clearly and systematically decompose sigma into its constituent parts and then track how each of these is treated within a PSHA, it is vital to adopt a clear and consistent nomenclature. Ambiguity of definitions and misuse of terminology are endemic to the field of seismic hazard study (e.g., Abrahamson, 2000), and it is vitally important to address this if the state-of-practice is to be brought into line with the state-of-the-art. To date, there seems to be no consensus on the notations used to refer to the variability of ground-motion models and its various components.

In this paper, we provide a clear description of the variability of ground-motion models on rock and soil sites and their various components. The notations given in this paper will be used in the development of ground-motion models for the Next Generation Attenuation for the Central and Eastern United States (NGA-East) project and for the PEGASOS Refinement project (Renault et al., 2010), and it is the hope of the authors that they will be widely adopted for use in this field. The acronyms and indices which we are using are defined with the goal to be more or less self-explanatory. A capital B for example stands for “between”, a capital W for “within” and the use of “2” in the standard deviation notation suggests that the corresponding variability is treated as an epistemic uncertainty if a non-ergodic assumption is applied. Moreover, subscripts e and s are the earthquake and site indices, respectively, subscript l is the earthquake location index, and superscripts B and G refer to baserock and ground surface, respectively. This way, we use only a minimum number of Greek symbols. For clarity, a glossary of terms presenting our proposed terminology and symbols is provided. Table A, given in the Appendix of the paper, provides a
correspondence between the new terminology and those that have been used in various other studies.

Ground-Motion Models

In this section, we return to fundamentals and discuss the basic formulation of GMPEs in order to introduce our preferred nomenclature and to clearly define each component of the models and their associated variability. An empirical GMPE generally has the general form:

\[ Y = f(X_{es}, \theta) + \Delta \]  

where \( Y \) is the natural logarithm of the observed ground-motion parameter, \( f(X_{es}, \theta) \) is the ground-motion model, \( X_{es} \) is the vector of explanatory parameters (e.g., magnitude, distance, style of faulting, site conditions), \( \theta \) is the vector of model coefficients and \( \Delta \) is a random variable describing the total variability of the ground motion. \( \Delta \) is usually decomposed into between-events variability, \( \Delta B \), and within-event variability, \( \Delta W \), which are zero-mean, independent, normally-distributed random variables with standard deviations \( \tau \) and \( \phi \), respectively. As illustrated in Figure 1, the between-events residual (also called inter-event residual or event term), \( \delta B \), represents the average shift of the observed ground motion from an individual earthquake, \( e \), from the population median predicted by the ground-motion model. The within-event residual (also called intra-event residual), \( \delta W \), is the misfit between an individual observation at station \( s \) from the earthquake-specific median prediction, which is defined as the median prediction of the model plus the between-event term for earthquake \( e \). The between-events and within-event standard deviations of the ground-motion model represent the earthquake-to-earthquake variability and record-to-record variability, respectively. The between-events and within-event residuals are uncorrelated, so the total standard deviation of the ground-motion model, \( \sigma \), can be written as:

\[ \sigma = \sqrt{\tau^2 + \phi^2} \]  

To further analyze and decompose the residuals and variances of ground-motion models into their respective components, consider the sketch of a site with soil overlying rock as shown in Figure 2. The observed ground motion on baserock at point \( B \) and at period \( T \) can be written as:

\[ y_{es}^B(T) = \mu_{es}^B(M_e, R_{es}, F_e, ZtoR_e, X_{es}^B, T) + \delta B_{es}^B + \delta W_{es}^B \]
Figure 1. Between-event and within-event components of ground-motion variability (after Strasser et al., 2009)

Figure 2. Site response reference points
where \( \gamma_{es}^B \) is the natural logarithm of the observed ground-motion parameter on baserock at site \( s \) during earthquake \( e \), \( \mu_{es}^B \) is the predicted median ground motion on baserock for an earthquake of magnitude \( M_e \), style-of-faulting \( F_e \), and depth-to-top of rupture \( ZtoR_e \) at site \( s \) with site parameter \( X^B_s \) (e.g., site class, shear wave velocity in the upper 30 m of the site profile) located at a rupture distance \( R_e \); \( \delta B_{\mu e}^B \) and \( \delta W_{\mu e}^B \) are the corresponding between-events and within-event residuals on baserock. The between-events and within-event residuals on baserock have standard deviations \( \tau^B \) and \( \phi^B \), respectively.

The observed ground motion on the ground surface at point \( G \) and at period \( T \) can be written as:

\[
y_{es}^G (T) = \mu_{\mu e}^G (M_e, R_e, F_e, ZtoR_e, X^G_s, T) + \delta B_{\mu e}^G + \delta W_{\mu e}^G
\]

where \( y_{es}^G \) is the natural logarithm of the observed ground-motion parameter on the ground surface at station \( s \) during earthquake \( e \), \( \mu_{\mu e}^G \) is the corresponding predicted median ground motion on soil, and \( X^G_s \) is the vector of site parameters (e.g., site class, shear wave velocity in the upper 30 m of the site profile, depth of soil). The between-events and within-event residuals on ground surface, \( \delta B_{\mu e}^G \) and \( \delta W_{\mu e}^G \), are part of zero-mean, normal distributions with standard deviations \( \tau^G \) and \( \phi^G \), respectively.

The between-events residual represents average source effects (averaged over all azimuths) and reflect the influence of factors such as stress drop and variation of slip in space and time that are not captured by the inclusion of magnitude, style-of-faulting and source depth. The within-event residual represents azimuthal variations in source, path, and site effects reflecting the influence of those factors such as crustal heterogeneity, deeper geological structure and near-surface layering that are not captured by a distance metric and a site-classification based on the average shear-wave velocity.

The computed sigma from empirical ground-motion models includes a contribution from measurement errors in the determination of the explanatory variables in the models. The influence of such metadata uncertainties can be quantified and removed from the models. Strasser \textit{et al.} (2009) give examples of reductions of sigma to account for measurement errors in magnitude, distance, depth, and other parameters; in all cases, the reduction in sigma due to measurement errors is modest.
Components of the Variability of Ground-Motion Models

The distinction between within-event and between-events variability is very useful for quantifying, understanding, and handling the ground-motion variability and addressing the correlation of the residuals. To reduce the value of sigma, however, it is first necessary to decompose the variability into smaller parts. The within-event residual of ground-motion models include systematic baserock or site-specific effects and path-specific effects. Similarly, the between-events residual contains systematic source-specific effects. Removing these systematic effects is a key to removing the ergodic assumption from the seismic hazard and requires repeated sampling of the site, path, and source in question.

According to Walling (2009), estimating the various components of the variability of ground-motion models depends on the type of dataset available for the ground-motion regression. Walling (2009) describes five types of ground-motion datasets. The most commonly used types of datasets are:

1. A global dataset contains recordings of ground motion at multiple sites from earthquakes in multiple regions. Such datasets were used in the development of the Next Generation Attenuation models for Western United States (NGA-West) assuming that ground motion is similar across all regions and sites for one tectonic class.

2. A site-specific dataset contains multiple recordings at one site from earthquakes located in different source regions and can be used to estimate the systematic and repeatable site-specific effects.

3. A path-specific dataset contains multiple recordings at one site from earthquakes located in a small source region. It can be used to estimate the source region, site, and path effects on the ground-motion model.

If downhole data at the site are available (site-specific dataset on baserock), we can estimate the baserock-specific effects on the ground-motion model. The different components of the between-events and within-event residuals and standard deviations of ground-motion models are described below.

Components of the Within-Event Variability

The ground motion at the soil surface at point $G$ is the product of the input rock ground motion at point $B$ and the site amplification factor. Assuming linear site response, the natural logarithm of the ground motion at point $G$ can be written as:

$$y^G_m(T) = y^B_m(T) + \ln Amp_m(T, X_s)$$  \hspace{1cm} (5)
where \(Amp_{es}(T, X_s)\) is the site amplification factor at station \(s\) for earthquake \(e\) and \(X_s\) is the vector of explanatory site parameters. The observed site amplification at station \(s\) for earthquake \(e\) can be broken down into the median amplification factor for the simple site classification to which site \(s\) is assumed to belong, \(\mu_{es}^\text{amp}(T, X_s)\), a site-to-site residual, \(\delta S 2S_s\), and a site amplification residual, \(\delta Amp_{es}\), as follows:

\[
\ln Amp_{es}(T, X_s) = \mu_{es}^\text{amp}(T, X_s) + \delta S 2S_s + \delta Amp_{es}
\]

(6)

Given multiple recordings at an individual site (site-specific dataset on soil surface), the site-to-site residual, \(\delta S 2S_s\), represents the systematic deviation of the observed amplification at this site from the median amplification predicted by the model using simple site classification such as the average shear-wave velocity in the uppermost 30 meters at the site, \(V_{\text{S30}}\). The site-to-site residual results from the use of simple site parameters which do not provide a complete site characterization. For example, if \(V_{\text{S30}}\) is used for sites classification, two sites with the same \(V_{\text{S30}}\) can still have significantly different site profiles and therefore have different site amplifications. This difference in the site amplification caused by having sites with the same \(V_{\text{S30}}\) but different profiles is reflected in the site-to-site residual. The remaining site amplification residual, \(\delta Amp_{es}\), describes the record-to-record variability of the amplification at site \(s\) for earthquake \(e\).

It is the misfit between an individual observation of the amplification at site \(s\) due to earthquake \(e\) from the average site-specific amplification and is caused by effects such as variability in incidence angle, 3D structure, or variability of input waveform (phasing) of rock motion. The site-to-site and site amplification standard deviations are \(\phi_{S2S}\) and \(\phi_{Amp}\), respectively; \(\phi_{S2S}\) represents site-to-site variability within a site class, while \(\phi_{Amp}\) represents unexplained variability in the site amplification at an individual site.

The within-event residual on baserock, \(W_{el}^\text{b}\), includes systematic wave propagation effects. If we have multiple recordings at one site from a small source region (single-path dataset), then we can separate these systematic path effects from the within-event residuals as follows:

\[
\delta W_{el}^\text{b} = \delta P 2P_{el} + \delta W_{el}^\text{b}
\]

(7)

where \(\delta P 2P_{el}\) is the path-to-path residual at site \(s\) for source region \(l\) and \(\delta W_{el}^\text{b}\) is the remaining within-event residual for earthquake \(e\) at site \(s\) and in source region \(l\). The path-to-path residual represents the average shift of the observed site-specific region-specific ground motion from the median site-specific model prediction and has a standard deviation of \(\phi_{P2P}\). In other words, it represents how the specific characteristics of this travel path lead to motions that are systematically different from the average obtained from the model. The within-event residual,
\( \delta W_{es}^0 \), represents the remaining unexplained path and radiation pattern effects and has a standard deviation of \( \phi_{0,G} \).

In this paper, we assume linear site response; the effects of soil nonlinearity on the standard deviations of ground-motion models are discussed in Al Atik and Abrahamson (2010). Combining the previous equations and assuming linear site response, the natural logarithm of the median ground motion on soil at point \( G \) and the intra-event residual on soil surface can be written as:

\[
\mu_{es}^G (M_e, R_e, F_e, ZtoR_e, X_e^G) = \mu_{es}^B (M_e, R_e, F_e, ZtoR_e, X_e^B) + \mu_{amp}^e (T, X_e) \tag{8}
\]

\[
\delta W_{es}^G = \delta S2S_e + \delta Amp_e + \delta P2P_{sl} + \delta W_{cil}^0 \tag{9}
\]

If downhole data is available for baserock \( b \) underlying site \( s \), the systematic baserock-specific effects can be estimated and the within-event residual on baserock can be broken down as follows:

\[
\delta W_{es}^B = \delta B2B_{sb} + \delta P2P_{sl} + \delta W_{cil}^0 \tag{10}
\]

where \( \delta B2B_{sb} \) is the baserock-to-baserock residual for baserock \( b \) underlying site \( s \) and \( \delta W_{cil}^0 \) is the within-event residual for earthquake \( e \) located in source region \( l \) and at site \( s \) overlaying baserock \( b \). The baserock-to-baserock residual represents the average shift of the observed input baserock-specific ground motion from the median baserock ground motion predicted by the global model (for that magnitude and distance) and has a standard deviation of \( \phi_{B2B} \). The path-specific within-event residual on baserock, \( \delta W_{cil}^0 \), represents all remaining effects not included in the ground-motion model and has a standard deviation of \( \phi_{0,B} \).

We assume that the components of the within-event and between-events residuals are uncorrelated. Additional work is needed to test the correlation of the various variability components. The within-event standard deviations on baserock and on ground surface, \( \phi^B \) and \( \phi^G \), respectively can be written as:

\[
\phi^B = \sqrt{\phi_{B2B}^2 + \phi_{P2P}^2 + \phi_{0,B}^2} \tag{11}
\]

\[
\phi^G = \sqrt{\phi_{S2S}^2 + \phi_{Amp}^2 + \phi_{P2P}^2 + \phi_{B,G}^2} \tag{12}
\]
Components of the Between-Events Variability

The between-events residual on baserock, $\delta B^e_{\text{b}}$, describes the deviation of the source properties of earthquake $e$ from the average source properties of the earthquakes in the global dataset. The between-events residual on baserock contains systematic source effects that can be removed if we have multiple recordings from a single source region (path-specific dataset). In such case, the between-events residual on baserock can be written as:

$$\delta B^e_{\text{b}} = \delta L2L_l + \delta B^0_{el}$$

(13)

where $\delta L2L_l$ is the location-to-location residual for earthquakes in source region $l$ and has a standard deviation of $\tau_{L2L}$. It represents the average shift in the ground motion for an earthquake in a single region as compared to the median predicted by the global ground-motion model. The reason for such a shift could be, for example, a lower or higher than the average stress drop for that particular source region. $\delta B^0_{el}$ is the remaining between-events residual for earthquake $e$ after removing the earthquake location-specific effects and has a standard deviation of $\tau_0$.

Assuming linear site response and uncorrelated components of the between-events residual, the between-events residual on baserock and ground surface, $\tau^b$ and $\tau^G$, respectively are:

$$\tau^G = \tau^b = \sqrt{\tau_{L2L}^2 + \tau_0^2}$$

(14)

The total standard deviation of the global ground-motion model on baserock, $\sigma^B$, and on ground surface, $\sigma^G$, when applying the ergodic assumption can be written as:

$$\sigma^B = \sqrt{\phi_{B2B}^2 + \phi_{P2P}^2 + \phi_{0,B}^2 + \tau_{L2L}^2 + \tau_0^2}$$

(15)

$$\sigma^G = \sqrt{\phi_{S2S}^2 + \phi_{Amp}^2 + \phi_{P2P}^2 + \phi_{0,G}^2 + \tau_{L2L}^2 + \tau_0^2}$$

(16)

Aleatory Variability and Epistemic Uncertainty

Seismic hazard studies incorporate two types of uncertainty: aleatory variability and epistemic uncertainty. By definition, aleatory variability is the natural randomness in a process that cannot be reduced with increasing knowledge about the process. Epistemic uncertainty is the scientific uncertainty in the model of the process; it is caused by limited data and knowledge and is characterized by alternative models. With increased data and knowledge, the epistemic uncertainty can, in theory, be reduced to zero.
In the context of the definitions presented here, the different components of the variability of empirical ground-motion models with and without the ergodic assumption can be characterized as aleatory variability or epistemic uncertainty as shown in Table 1 for common application scenarios.

**Table 1. Characterizing the different components of the variability of empirical ground-motion models for common application scenarios with and without the ergodic assumption**

<table>
<thead>
<tr>
<th>Aleatory</th>
<th>Epistemic</th>
<th>Required Dataset</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ergodic (on soil surface)</strong></td>
<td>$\phi_{0,G}$, $\tau_0$, $\phi_{Amp}$, $\phi_{S2S}$, $\phi_{P2P}$, $\tau_{L2L}$</td>
<td>Global: Multiple recordings at different sites (on soil surface) from earthquakes in multiple source regions</td>
</tr>
<tr>
<td><strong>Fully non-ergodic (single-path on soil surface)</strong></td>
<td>$\phi_{0,G}$, $\phi_{Amp}$, $\tau_0$</td>
<td>Path-specific on soil surface: Multiple recordings at one site (on soil surface) from earthquakes in one location</td>
</tr>
<tr>
<td><strong>Partly non-ergodic (single-site on soil surface)</strong></td>
<td>$\phi_{P2P}$, $\phi_{0,G}$, $\phi_{Amp}$, $\tau_{L2L}$, $\tau_0$</td>
<td>Site-specific: Multiple recordings at one site (on soil surface) from earthquakes located in different source regions</td>
</tr>
<tr>
<td><strong>Partly non-ergodic (single-site on baserock)</strong></td>
<td>$\phi_{P2P}$, $\phi_{0,B}$, $\tau_{L2L}$, $\tau_0$</td>
<td>Baserock-specific: Multiple recordings at one site (on baserock) from earthquakes located in different source regions</td>
</tr>
</tbody>
</table>

As discussed in the previous section, the site-to-site, path-to-path, baserock-to-baserock, and earthquake location-to-location variability represent systematic effects that can be removed from the variability of the ground motion if we have multiple path-specific recordings on baserock or on soil surface. Under fully non-ergodic assumption (single-path scenario), the site-to-site, path-to-path, baserock-to-baserock and earthquake location-to-location standard deviations are, therefore, epistemic uncertainties. If we develop more elaborate models that better characterize the complex nature of these processes, these uncertainties can be reduced. The total aleatory variability of single-path ground motion on soil surface, $\sigma_{GSP}$, and on baserock, $\sigma_{BSP}$, can be written as:
When a partial non-ergodic assumption that only removes the systematic site-specific effects is applied to the ground motion on soil (single-site GMPE), the site-to-site variability is an epistemic uncertainty and the single-site within-event residual at station \( s \) for earthquake \( e \), \( \delta W_{s_e} \), can be defined as:

\[
\delta W_{s_e} = \delta W_e - \delta S 2S_s
\]  

(19)

The standard deviation of \( \delta W_{s_e} \) is called single-station within-event standard deviation and is referred to as \( \phi_{ss} \). The aleatory variability of single-site ground motion, \( \sigma_{ss} \), can be written as:

\[
\sigma_{ss} = \sqrt{\phi_{ss}^2 + \tau^2} = \sqrt{\phi_{amp}^2 + \phi_{P2P}^2 + \phi_{0,G}^2 + \tau_{L2L}^2 + \tau_0^2}
\]  

(20)

Similarly, applying a partial non-ergodic assumption that only removes the systematic baserock-specific effects from the variability of the ground motion on baserock, the baserock-to-baserock variability becomes an epistemic uncertainty and the aleatory variability of single-baserock ground motion, \( \sigma_{sb} \), is:

\[
\sigma_{sb} = \sqrt{\phi_{P2P}^2 + \phi_{0,B}^2 + \tau_{L2L}^2 + \tau_0^2}
\]  

(21)

In deterministic and probabilistic hazard studies that apply the ergodic assumption, the total standard deviation of the empirical ground-motion model on baserock or on the soil surface, \( \sigma^B \) or \( \sigma^G \) respectively, is assumed to be an aleatory variability. All components of the variability are classified as aleatory variability when applying the ergodic assumption.

**Application to Hazard Studies**

Removing the ergodic assumption in seismic hazard analyses leads to a large reduction in the aleatory variability of ground-motion models. Tables 2 and 3 present a comparison of the standard deviations of ground motion models obtained in previous studies with and without the ergodic assumption for peak ground acceleration, PGA, and spectral acceleration at a period of 1 second, SA(1.0). Tables 2 and 3 show significant reduction in the aleatory variability of the ground motion for PGA and SA(1.0) with partially non-ergodic assumption (single-site or single-
baserock ground motions). A much larger reduction is observed when applying the fully non-ergodic assumption.

The removal of the ergodic assumption also impacts the median of ground-motion models. Under a non-ergodic assumption, we can no longer use median ground-motion values predicted by models developed from a global dataset to site-specific or path-specific scenarios. The non-ergodic assumption requires adding epistemic uncertainty in the median ground motion for each site/source combination. For single-site GMPEs, epistemic uncertainty in the site amplification is required. This is commonly treated in site-specific site-response studies conducted in major projects. Applying fully non-ergodic assumption is more complicated because it requires epistemic uncertainty to be added for every site-path combination and because the epistemic uncertainty in the median ground motion will be correlated for closely spaced sources. The spatial correlation in the systematic and repeatable effects needs to be addressed if fully non-ergodic assumption is to be applied to probabilistic hazard studies (e.g., Walling, 2009).

| Table 2. Total standard deviation (natural logarithms) of PGA from previous studies |
|-----------------------------------------------|---------------------|---------------------|---------------------|
|                                              | On Ground Surface   | On Baserock         |
|                                              | $\sigma_G^G$       | $\sigma_{SS}^G$    | $\sigma_{SP}^G$    |
|                                              | $\sigma_{SB}^B$    | $\sigma_{SB}^B$    | $\sigma_{SP}^B$    |
| Atkinson (2006)                             | 0.711              | 0.617              | 0.414              |
|                                              | -                  | -                  | -                  |
| Chen & Tsai (2002)                          | 0.731              | 0.631              | -                  |
|                                              | -                  | -                  | -                  |
| Morikawa et al. (2008)                      | 0.780              | -                  | 0.360              |
|                                              | -                  | -                  | -                  |
| Lin et al. (2010)                           | 0.680              | 0.619              | 0.365              |
|                                              | -                  | -                  | -                  |
| Rodriguez-Marek et al. (2009)               | 0.816              | 0.634              | -                  |
|                                              | 0.719              | 0.626              | -                  |
Table 3. Total standard deviation (natural logarithms) of SA(1.0) from previous studies

<table>
<thead>
<tr>
<th></th>
<th>On Ground Surface</th>
<th>On Baserock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma^G$</td>
<td>$\sigma_{SS}$</td>
</tr>
<tr>
<td>Atkinson (2006)</td>
<td>0.668</td>
<td>0.617</td>
</tr>
<tr>
<td>Chen &amp; Tsai (2002)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Morikawa et al. (2008)</td>
<td>0.909</td>
<td>-</td>
</tr>
<tr>
<td>Lin et al. (2010)</td>
<td>0.741</td>
<td>0.636</td>
</tr>
<tr>
<td>Rodriguez-Marek et al. (2009)</td>
<td>0.804</td>
<td>0.6245</td>
</tr>
</tbody>
</table>

Conclusions

The identification of the various components of ground-motion variability is promising for the improvement of PSHA since some of the apparent randomness can be transformed to epistemic uncertainty, and with additional data this uncertainty can be removed. However, if it is not removed through the acquisition of additional information (recorded data or numerical simulations), the increased epistemic uncertainty must be included in the logic tree. If there is no additional data to constrain the epistemic uncertainty, the consequence will be that the mean hazard curve remains unchanged and the fractile hazard curves will be broadened. This approach is still useful for showing the different components of the uncertainty and setting a framework for future work to reduce this uncertainty.

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opinions, findings, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the funding or sponsoring agencies.

**Glossary of Terms**

\[ Y : \] Natural logarithm of observed ground motion

\[ T : \] Spectral period (in seconds)

\[ y_{es}^B : \] Natural logarithm of the observed ground motion on baserock at station \( s \) during earthquake \( e \)

\[ y_{es}^G : \] Natural logarithm of the observed ground motion on ground surface at station \( s \) during earthquake \( e \)

\[ f(X_{es}, \theta) : \] Ground-motion model

\[ \theta : \] Vector of model coefficients

\[ X_{es} : \] Vector of explanatory parameters (e.g., magnitude, distance, style of faulting, site conditions)

\[ X_s : \] Vector of explanatory site parameters

\[ \mu_{es}^B : \] Predicted median ground motion on baserock at station \( s \) for earthquake \( e \)

\[ \mu_{es}^G : \] Predicted median ground motion on ground surface at station \( s \) for earthquake \( e \)

\[ Amp_{es} (T, X_s) : \] Site amplification factor at station \( s \) for earthquake \( e \)

\[ \mu_{es}^{Amp} (T, X_s) : \] Median site amplification factor

\[ \Delta : \] Random variable describing the total variability of the ground motion

\[ \Delta B : \] Random variable describing the between-events variability of the ground motion

\[ \Delta W : \] Random variable describing the within-event variability of the ground motion

\[ \delta B_e : \] Between-events residual for earthquake \( e \)

\[ \delta W_{es} : \] Within-event residual at station \( s \) for earthquake \( e \)

\[ \delta WS_{es} : \] Single-station within-event residual at station \( s \) for earthquake \( e \)
\( \delta B^b_e \): Between-events residual on baserock for earthquake \( e \)

\( \delta W^b_{es} \): Within-event residual on baserock at station \( s \) for earthquake \( e \)

\( \delta B^g_e \): Between-events residual on ground surface for earthquake \( e \)

\( \delta W^g_{es} \): Within-event residual on ground surface at station \( s \) for earthquake \( e \)

\( \delta S2S_s \): Site-to-site residual for site \( s \)

\( \delta Amp_{es} \): Unexplained site amplification residual for station \( s \) and earthquake \( e \)

\( \delta P2P_{sl} \): Path-to-path residual for site \( s \) and source region \( l \)

\( \delta B2B_{sb} \): Baserock-to-baserock residual for baserock \( b \) underlying site \( s \).

\( \delta L2L_l \): Earthquake location-to-location residual for source region \( l \)

\( \delta W^0_{esl} \): Unexplained within-event residual for earthquake \( e \), site \( s \) and source region \( l \)

\( \delta W^0_{eslb} \): Unexplained within-event residual for earthquake \( e \) at site \( s \) with baserock \( b \) and in source region \( l \)

\( \delta B^0_{es} \): Unexplained between-events residual for earthquake \( e \) and source region \( l \)

\( \tau \): Standard deviation of the between-events residual

\( \phi \): Standard deviation of the within-event residual

\( \tau^b \): Standard deviation of the between-events residual on baserock

\( \phi^b \): Standard deviation of the within-event residual on baserock

\( \tau^g \): Standard deviation of the between-events residual on ground surface

\( \phi^g \): Standard deviation of the within-event residual on ground surface

\( \phi_{S2S} \): Standard deviation of the site-to-site residual

\( \phi_{S2S} \): Single-station within-event standard deviation

\( \phi_{Amp} \): Standard deviation of the unexplained site amplification residual

\( \phi_{P2P} \): Standard deviation of the path-to-path residual
\( \phi_{BB} \): Standard deviation of the baserock-to-baserock residual

\( \tau_{LL} \): Standard deviation of earthquake location-to-location residual

\( \phi_{0,B} \): Standard deviation of the within-event single path residual on baserock

\( \phi_{0,G} \): Standard deviation of the within-event single path residual on ground surface

\( \tau_{0} \): Standard deviation of the between-events, single source-region residual

\( \sigma^B \): Aleatory variability of the ground-motion model on baserock under ergodic assumption

\( \sigma^G \): Aleatory variability of the ground-motion model on soil surface under ergodic assumption

\( \sigma^G_{SP} \): Aleatory variability of the ground-motion model on soil surface under full non-ergodic assumption (single-path on soil surface)

\( \sigma^B_{SP} \): Aleatory variability of the ground-motion model on baserock under full non-ergodic assumption (single-path on baserock)

\( \sigma_{SS} \): Aleatory variability of the ground-motion model on soil surface under partial non-ergodic single-site assumption

\( \sigma_{SB} \): Aleatory variability of the ground-motion model on baserock under partial non-ergodic single-baserock assumption

References


Al Atik, L. and N. A. Abrahamson (2010). Nonlinear site response effects on the standard deviations of predicted ground motions (submitted for publication).


## Appendix

Table A. Correlation of notation differences for components of the variability of ground-motion models from previous studies with the current study

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<tr>
<td>Total on soil surface</td>
<td>$\sigma^G$</td>
<td>$\sigma^I$</td>
<td>$\sigma_T$</td>
<td>$\sigma_{\text{reg}}$</td>
<td>$\delta$</td>
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<tr>
<td>Total on baserock</td>
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<tr>
<td>Between-events</td>
<td>$\tau$</td>
<td>$\sigma_{\text{events}}$</td>
<td>$\tau$</td>
<td>$\sigma_E$</td>
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<td>$\tau$ (no correction)</td>
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<tr>
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<td>$\phi$</td>
<td>$\sigma^{\text{IE}}$</td>
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<td>$\sigma$ (no correction)</td>
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<td>Site-to-site</td>
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<tr>
<td>Path-to-path</td>
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<td>$\tau_{SR}$</td>
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<td>Within-event, single-path on soil surface</td>
<td>$\sqrt{\phi_{\text{Amp}}^2 + \phi_{0,G}^2}$</td>
<td>$\sigma^R$</td>
<td>$\sigma_0$</td>
<td>$\sigma$ (with correction)</td>
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<td>$\sigma_\tau$</td>
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<td>$\sigma_{SS}^{IV}$</td>
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<td>$\sigma_{le}$</td>
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<td>$\sqrt{\sigma^2 + \tau^2}$ (with correction)</td>
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